Int J Theor Phys (2009) 48: 3629–3634
DOI 10.1007/s10773-009-0170-3

Higher Order Corrections of 2D Gravity on AdS_2

J. Sadeghi · B. Pourhassan

Received: 22 July 2009 / Accepted: 8 October 2009 / Published online: 20 October 2009

Abstract In this paper we consider 2D Maxwell-dilaton gravity and calculate modified entropy due to higher order terms of the form $\sum a_n R^n$. We find that, higher order correction decreases the value of the black hole entropy. Then we compare the effect of higher order corrections with effect of Chern-Simons term on the black hole entropy and obtain the expansion coefficient a_n.

Keywords Entropy function · 2D gravity · Black hole

1 Introduction

The entropy function formulation of Sen [1], which derived from Wald's formula [2] is the good method to calculate the black hole entropy, specially in two dimension [3–5]. In that case the possibility of two dimensional black holes by means of a conformal field theory has been investigated [6–12]. In two dimensional dilaton gravities [13–15] it has been shown that, the entropy is proportional to the value of the dilaton field at the horizon. In order to find the black hole entropy, by using entropy function method [16–23], at the first, one can rewrite the lagrangian density in terms of the near horizon value of fields, and then take the Legendre transformation of the resulting function with respect to the electric field, and finally multiply with an overall factor 2π. Therefore the entropy function E is given by,

$$E = 2\pi[q_i \epsilon_i - \mathcal{L}], \tag{1}$$

where q_i denote electric charges, and ϵ_i are near horizon radial electric field. The entropy function E is a function of the electric charges q_i and various parameters of near horizon

J. Sadeghi · B. Pourhassan (✉)
Sciences Faculty, Department of Physics, Mazandaran University, P.O. Box 47415-416, Babolsar, Iran
e-mail: b.pourhassan@umz.ac.ir

J. Sadeghi
e-mail: pouriya@ipm.ir

background, such as sizes of AdS_2 space. These parameters obtained by extremizing E with respect to them. Finally, the black hole entropy is given by the value of E at its extremum.

Already, in Ref. [24], author studied 2D Maxwell-dilaton gravity with higher order corrections given by the Chern-Simons term. By using entropy function formalism they found the black hole entropy. Also they obtained the central charge and the level of $U(1)$ current. Now, in this article we consider 2D Maxwell-dilaton gravity in Sect. 2, and calculate modified black hole entropy due to higher order correction. Then, in Sect. 3 we compare our result with Ref. [24] and fix coefficients of higher order curvature terms.

2 2D Maxwell-Dilaton Gravity

In this section we consider two dimensional Maxwell-dilaton gravity and try to obtain the black hole entropy, by using entropy function formalism. Then we consider the effect of higher order corrections on the black hole entropy. The Einstein-Hilbert action of 2D Maxwell-dilaton gravity is given by,

$$S_{EH} = \frac{1}{8G}\int d^2x\sqrt{-g}e^{\phi}\left(R + 2\partial_\mu\phi\partial^\mu\phi + \frac{2}{L^2}e^{2\phi} - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu}\right), \tag{2}$$

where L is AdS_2 radius. The equations of motion are given by,

$$\begin{aligned}
&g_{\mu\nu}\left(\nabla^2 e^{\phi} + \frac{1}{L^2}e^{3\phi} - \frac{L^2}{4}F^2 e^{\phi} + e^{\phi}\partial_\mu\phi\partial^\mu\phi\right) - \nabla_\mu\nabla_\nu e^{\phi} - 2e^{\phi}\partial_\mu\phi\partial_\nu\phi = 0,\\
&R + \frac{6}{L^2}e^{2\phi} + \frac{L^2}{2}F^2 + 2e^{\phi}\partial_\alpha\phi\partial^\alpha\phi - 4\nabla^2 e^{\phi} = 0,\\
&\varepsilon^{\mu\nu}\partial_\mu(Fe^{\phi}) = 0,
\end{aligned} \tag{3}$$

where $F_{\mu\nu} = \sqrt{-g}\varepsilon_{\mu\nu}F$. The AdS_2 solution which preserves $SO(1,2)$ symmetry is given by the following ansatz,

$$\begin{aligned}
&ds^2 = v\left(-r^2dt^2 + \frac{1}{r^2}dr^2\right),\\
&e^{\phi} = u,\\
&F_{rt} = \frac{\epsilon}{L^2},
\end{aligned} \tag{4}$$

where u, v and ϵ are constants which can be determined in terms of the charge q. In order to obtain these parameters we extremize the entropy function with respect to them. By using solutions (4), the equation of motion (3) reduces to,

$$\begin{aligned}
&u^2 - \frac{\epsilon^2}{4v^2} = 0,\\
&\frac{\epsilon^2}{2v^2L^2} + \frac{6u^2}{L^2} - \frac{2}{v} = 0.
\end{aligned} \tag{5}$$

By eliminating ϵ from (5) we have the following condition,

$$4vu^2 = L^2. \tag{6}$$

Now, we are going to use Wald's formula to obtain the entropy function. In the firs step, by using solution (4) we should write the lagrangian density in terms of u, v and ϵ,

$$\mathcal{L}(u,v,\epsilon)=\frac{u}{8G}\left[-2+\frac{2vu^3}{L^2}+\frac{u\epsilon^2}{2vL^2}\right]. \tag{7}$$

Then, by using relation (1), one can obtain the following entropy function,

$$E(u,v,q)=\frac{\pi u}{2G}\left[1-\frac{uv^2}{L^2}\right]+8G\pi L^2q^2\frac{v}{u}, \tag{8}$$

where $q=\frac{\partial\mathcal{L}}{\partial\epsilon}=\frac{u\epsilon}{8GvL^2}$ denotes the electric charge carried by the black hole.

The third step is extremizing the entropy function with respect to u and v, which yield to the following equations,

$$\begin{aligned} v_e&=\frac{1}{16Gq},\\ u_e&=2L\sqrt{Gq}, \end{aligned} \tag{9}$$

where v_e and u_e are extremum values of v and u respectively. One can check that solutions (9) satisfy the condition (6). Then, we yield to the black hole entropy by rewriting the value of the entropy function (8) at the extremum,

$$S_{BH}(q)=\pi L\sqrt{\frac{q}{G}}. \tag{10}$$

Now, let us to consider the effect of the higher-derivative correction terms of the form R^n [1, 3]. In that case one should replace $\sum a_nR^n=R+a_2R^2+a_3R^3+\cdots$ instead R in the action (2). Hence the change of the action due to the higher-derivative correction is,

$$\Delta S_{EH}=\frac{1}{8G}\int d^2x\sqrt{-g}e^{\phi}\sum a_nR^n, \tag{11}$$

and,

$$\Delta\mathcal{L}=-\frac{uv}{8G}\sum a_n\left(\frac{2}{v}\right)^n. \tag{12}$$

Therefore one can obtain modified entropy function as the following equation,

$$E_{mod}(u,v,q)=2\pi\left[4GL^2q^2\frac{v}{u}-\frac{vu^3}{4GL^2}+\frac{vu}{8G}\sum a_n\left(\frac{2}{v}\right)^n\right], \tag{13}$$

and extremizing the modified entropy function $\frac{\partial E_{mod}}{\partial v}=0$ and $\frac{\partial E_{mod}}{\partial u}=0$ give us following relations respectively,

$$\frac{4Gq^2L^2}{u}-\frac{u^3}{4GL^2}+\frac{u}{8G}\sum(n-1)a_n\left(\frac{2}{v}\right)^n=0, \tag{14}$$

and,

$$\frac{4Gq^2L^2}{u^2}+\frac{3u^2}{4GL^2}-\frac{1}{8G}\sum a_n\left(\frac{2}{v}\right)^n=0. \tag{15}$$

From (14) and (15) one can obtain the following relations,

$$\begin{aligned} u^2 &= \frac{L^2}{2}\left[\frac{1}{6}\sum a_n\left(\frac{2}{v}\right)^n + \sqrt{\left(\frac{\sum a_n(\frac{2}{v})^n}{6}\right)^2 - \frac{64G^2q^2}{3}}\right], \\ &= \frac{L^2}{2}\left[\frac{1}{2}\sum (n-1)a_n\left(\frac{2}{v}\right)^n + \sqrt{\left(\frac{\sum (n-1)a_n(\frac{2}{v})^n}{2}\right)^2 + 64G^2q^2}\right], \end{aligned} \tag{16}$$

so, we find the following equation,

$$\begin{aligned} &\frac{256}{3}G^2q^2 + \frac{1}{6}\sum a_n\left(\frac{2}{v}\right)^n \sum (n-1)a_n\left(\frac{2}{v}\right)^n \\ &\quad + \frac{1}{4}\left[\sum (n-1)a_n\left(\frac{2}{v}\right)^n\right]^2 - \frac{1}{12}\left[\sum a_n\left(\frac{2}{v}\right)^n\right]^2 = 0. \end{aligned} \tag{17}$$

Therefore we obtain the modified entropy function as the following,

$$S_{mod} = \frac{\pi}{4G}u_e v_e \sum (2-n)a_n\left(\frac{2}{v}\right)^n, \tag{18}$$

where u_e given by (16) and v_e is root of (17). It is easy to check that $n = 1$ term of entropy (18) reduces to the entropy (10) if $a_1 = 1$. Also we see that the second term ($n = 2$) has no effect on the entropy, so one can rewrite the entropy (18) as the following form,

$$S_{mod} = \pi L\sqrt{\frac{q}{G}} - \frac{\pi}{4G}u_e \sum_{n=3}(n-2)2^n a_n v_e^{1-n}. \tag{19}$$

We see that, the higher order terms may be decrease the value of entropy (10), so all terms with $n > 2$ have negative effect on the entropy of black hole.

In the next section we consider higher order corrections given by the Chern-Simons term.

3 2D Gravity with Chern-Simons Corrections

In this section, we add the two-dimensional Chern-Simons term to the action (2), so the total action is given by $S = S_{EH} + S_{CS}$ where,

$$S_{CS} = -\frac{1}{32G\mu}\int d^2x(LR\varepsilon^{\mu\nu}F_{\mu\nu} + L^3\varepsilon^{\mu\nu}F_{\mu\rho}F^{\rho\sigma}F_{\sigma\nu}). \tag{20}$$

In that case, equations of motion (3) extended to the following relations,

$$\begin{aligned} &g_{\mu\nu}\left(\nabla^2 e^{\phi} + \frac{1}{L^2}e^{3\phi} - \frac{L^2}{4}F^2e^{\phi} + e^{\phi}\partial_\mu\phi\partial^\mu\phi\right) - \nabla_\mu\nabla_\nu e^{\phi} - 2e^{\phi}\partial_\mu\phi\partial_\nu\phi \\ &\quad - \frac{L}{2\mu}\left[g_{\mu\nu}\left(\nabla^2 F - L^2F^3 - \frac{R}{2}F\right) - \nabla\mu\nabla_\nu F\right] = 0, \\ &R + \frac{6}{L^2}e^{2\phi} + \frac{L^2}{2}F^2 + 2e^{\phi}\partial_\alpha\phi\partial^\alpha\phi - 4\nabla^2 e^{\phi} = 0, \\ &\varepsilon^{\mu\nu}\partial_\mu\left(Fe^{\phi} + \frac{1}{2\mu L}(R + 3L^2F^2)\right) = 0. \end{aligned} \tag{21}$$

Similar to the previous section one can find extremal value of v and u as following,

$$v_e = \frac{1}{16Gq}\left(1-\frac{1}{\mu L}\right),$$
$$u_e = 2L\sqrt{Gq}\left(1-\frac{1}{\mu L}\right)^{-\frac{1}{2}}. \tag{22}$$

Again, it is easy to check that solutions (22) satisfy equations of motion (21) under assumption (4). Now, the entropy of black hole may be written as the following expression [24],

$$S_{BH}(q) = \pi L\sqrt{\frac{q}{G}\left(1-\frac{1}{\mu L}\right)}. \tag{23}$$

It is clear that, in the case of $\mu \to \infty$ where the effect of the Chern-Simons term is zero, the entropy (23) reduces to the entropy (10).

If we assume that effect of the Chern-Simons term is small, then it may be interesting to expand the entropy (23),

$$S_{BH}(q) = \pi L\sqrt{\frac{q}{G}}\left(1-\frac{1}{2\mu L}-\frac{1}{8\mu^2L^2}-\frac{1}{16\mu^3L^3}+\mathcal{O}(\mu^{-4})\right). \tag{24}$$

Now we would like to compare order by order the correction term in relation (24) and relation (10), thus one can obtain,

$$a_3 = \frac{G}{4\pi}\frac{v_e}{u_e}\frac{v_e}{\mu L},$$
$$a_4 = \frac{G}{4\pi}\frac{v_e}{16u_e}\left[\frac{v_e}{\mu L}\right]^2, \tag{25}$$
$$a_5 = \frac{G}{4\pi}\frac{v_e}{96u_e}\left[\frac{v_e}{\mu L}\right]^3,$$

however, about expansion coefficient we find that $a_1 = 1$, $a_2 = 0$ and

$$a_n \sim \frac{v_e}{u_e}\left[\frac{v_e}{\mu L}\right]^{n-2} \tag{26}$$

with $n \geq 3$.

4 Conclusion

In this paper we considered 2D Maxwell-dilaton gravity, and by using entropy function formalism calculated modified entropy, due to higher order correction. In this method one can consider higher order correction by expansion $\sum a_n R^n$. On the other hand, the effect of higher order corrections calculated by adding Chern-Simons term to the Einstein-Hilbert action of 2D Maxwell-dilaton gravity [24]. We compared results of these methods and obtained expansion coefficient. We have shown that the second order term have no effect on the entropy of black hole, and the other terms have negative effect on the black hole entropy proportional to $\frac{v_e}{u_e}[\frac{v_e}{\mu L}]^{n-2}$, with $n \geq 3$.

References

1. Sen, A.: J. High Energy Phys. **0509**, 038 (2005)
2. Wald, R.M.: Phys. Rev. D **48**, 3427 (1993)
3. Sadeghi, J., Setare, M.R., Pourhassan, B.: Two dimensional black hole entropy. Eur. Phys. J. C **53**, 95–97 (2008)
4. Sadeghi, J., Setare, M.R., Pourhassan, B.: Entropy of extremal black holes in two dimension. Acta Phys. Pol. B **40**(2), 251 (2009). arxiv:0707.0420v1 [hep-th]
5. Hyun, S., Kim, W., Oh, J.J., Sona, E.J.: Entropy function and universal entropy of two-dimensional extremal black holes. J. High Energy Phys. **0704**, 057 (2007)
6. Cadoni, M., Mignemi, S.: Phys. Rev. D **59**, 081501 (1999)
7. Cadoni, M., Mignemi, S.: Nucl. Phys. B **557**, 165 (1999)
8. Strominger, A.: J. High Energy Phys. **01**, 007 (1999)
9. Cacciatori, S., Klemm, D., Zanon, D.: Class. Quantum Gravity **17**, 1731 (2000)
10. Cadoni, M., Cavagli'a, M.: Phys. Rev. D **63**, 084024 (2001)
11. Cadoni, M., Cavagli'a, M.: Phys. Lett., Sect. B **499**, 315 (2001)
12. Cadoni, M., Carta, P.: Phys. Lett., Sect. B **522**, 126 (2001)
13. Callan, C.G., Giddings, S.B., Harvey, J.A., Strominger, A.: Phys. Rev. D **45**, 1005 (1992)
14. Jackiw, R.: Nucl. Phys., Sect. B **252**, 343 (1985)
15. Teitelboim, C.: Phys. Lett., Sect. B **126**, 41 (1983)
16. Silva, P.J.: Euclidean methods and the entropy function. Fortschr. Phys. **56**, 856–861 (2008)
17. Cho, J.-H., Ko, Y., Namæ, S.: The entropy function for the extremal Kerr-(anti-)de Sitter black holes. arXiv:0804.3811 [hep-th]
18. Cardoso, G.L., de Wit, B., Mahapatra, S.: Black hole entropy functions and attractor equations. J. High Energy Phys. **0703**, 085 (2007)
19. Cai, R.-G., Cao, L.-M.: On the entropy function and the attractor mechanism for spherically symmetric extremal black holes. Phys. Rev. D **76**, 064010 (2007)
20. Ge, X.-H., Shu, F.-W.: On black hole thermodynamics and the entropy function formalism. arxiv:0804.2123v2 [hep-th]
21. Alishahiha, M., Ebrahim, H.: Non-supersymmetric attractors and entropy function. J. High Energy Phys. **0603**, 003 (2006)
22. Garousi, M.R., Ghodsi, A., Houri, T., Khosravi, M.: More on entropy function formalism for non-extremal branes. J. High Energy Phys. **0903**, 026 (2009) arxiv:0812.4204v1 [hep-th]
23. Garousi, M.R., Ghodsi, A.: On attractor mechanism entropy function for non-extremal black holes/branes. J. High Energy Phys. **0705**, 043 (2007)
24. Alishahiha, M., Fareghbal, R., Mosaffa, A.E.: 2D gravity on AdS_2 with Chern-Simons corrections. J. High Energy Phys. **0901**, 069 (2009)